Chance

CHANCE

Series H

MATHLETICS

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Language of chance

Words are used nearly every day to describe the chance of an event occurring.

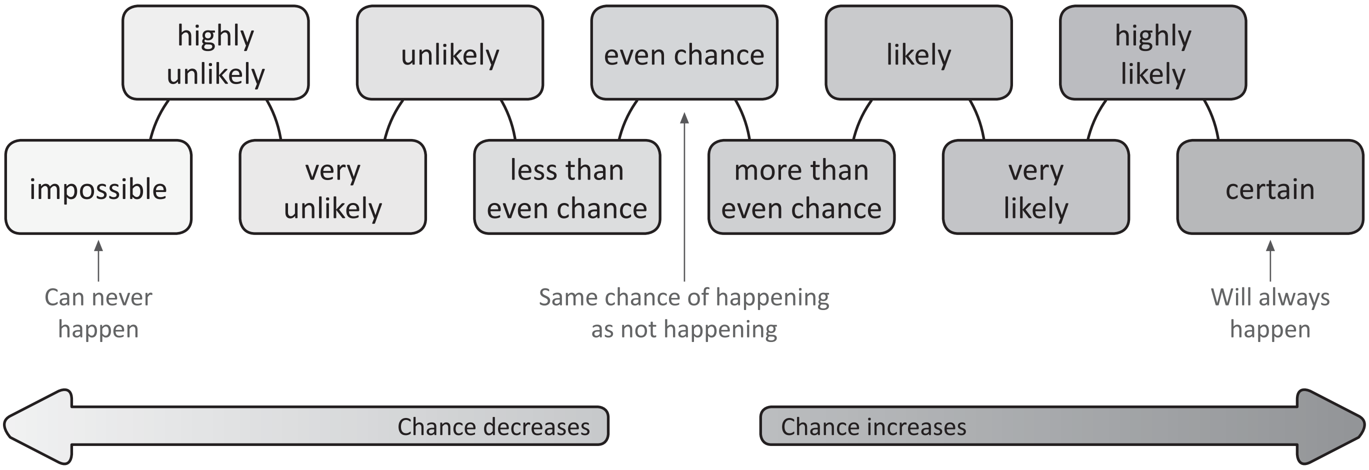
- The **event** is what we are observing to 'happen'

  "The weather report says it is **most likely** to **rain** sometime today"

  Chance description   Event

The words '**most likely**' imply that it **should** (or there is a good chance of) rain.

Here are some words that are often used to describe the chance or likelihood of events happening:



Use the words from the chart to describe the chance of these events

(i) Seeing a full moon on any given night during a calendar year?

A full moon occurs approximately 12 times a year, therefore **highly unlikely** chance.

(ii) Picking a red marble from a bag containing 49 blue and 51 red marbles.

There are slightly more red marbles than blue marbles, therefore **more than even chance**.

(iii) At least one student in an entire school blinking their eyes in the next hour.

Unless every student in the school can stare for very long periods, this is a **certain** chance.

(iv) Rolling a normal six-sided die and getting an even number.

There are three even and three odd numbers on a normal die, therefore there is an **even chance**.

Language of chance

1. Write down the words that best describe the normal everyday chance of these events happening:

a. Seeing a pink and yellow car on the road.

b. Picking a green piece of candy from a bag of all yellow candy.

c. Winning the lottery.

d. Saying the alphabet backwards in less than 5 seconds.

e. Not picking the correct cup out of three hiding a small ball.

f. Making an error when rushing through your work.

g. Picking a girl randomly out of a group of 10 girls and 10 boys.

h. Bumping into something walking around your home in the dark.

i. The Sun rising from the East.

j. Winning a raffle in which you buy all but one of the tickets.

k. Catching an orange fish from a tank of 8 white and 7 orange fish.

l. Opening a book to an odd numbered page at random.

m. An adventurer finds ice at the South Pole.

n. Rolling  on a normal six-sided die.

o. Winning a team game when your opponent has one less player.

p. Not beating a world record at a school sports carnival.

Sample space

When we randomly pick something from a group of objects, we are taking a small part (or sample).

A list of all the possible outcomes (results) from the group of objects is called the sample space.

Here a tennis ball, football, American football, basketball and a volleyball are being juggled.

If the juggler drops one, which ball (outcome) could it be?

The sample space for all outcomes = {tennis ball, football, American football, basketball, volleyball}

Separate each outcome by a comma inside curly brackets

Here are some more sample space examples relating to possible outcomes

(i) A bag contains positive integers less than 10. List the sample space when one number is picked at random.

The sample space for all the possible outcomes $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(ii) List the sample space of possible outcomes when this spinner is spun once.

The sample space of possible outcomes S = {★, ●, ■, ★, ▲, ♦, ✚, ⬟}

Write the outcome twice if repeated

(iii) What is the sample space of the outcomes when two coins are tossed?

For two events at the same time (i.e. tossing two coins), the sample space is all the possible pairings. A simple two-way table can be used to help ensure you get them all.

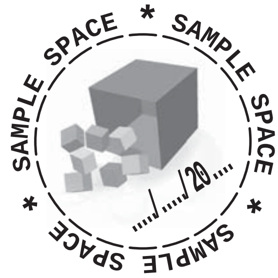
		Coin 2	
		Head (H)	Tail (T)
Coin 1	Head (H)	(H, H)	(H, T)
	Tail (T)	(T, H)	(T, T)

∴ The sample space for outcomes of two tossed coins

S = {(Head, Head), (Head, Tail), (Tail, Head), (Tail, Tail)}

S = {(H, H), (H, T), (T, H), (T, T)}

Sample space



1 Write down the sample space for all the possible outcomes for each of these:

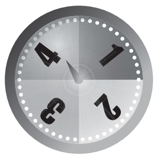
a Tossing a coin

$S =$

b Flicking a light switch

$S =$

c Spinning this spinner

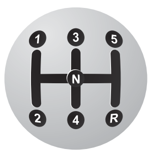


$S =$

d Rolling a normal six-sided dice

$S =$

e Moving this car gear stick



$S =$

f Pressing a key from this standard button keypad.

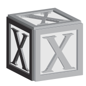
$S =$

g Choosing a letter from the word 'Mathematics'.

$S =$

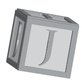
h A bag containing the letter blocks in the quantities shown here:

$\times 2$



$\times 4$

$\times 3$

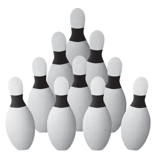


$\times 1$

$S =$

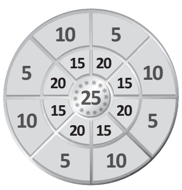
i The number of pins you can knock-over in one turn during a game of 10-pin bowling.

Psst: you might not knock over any pins!



$S =$

j The point scores possible with each dart thrown at this dart board.



$S =$

Sample space

2 Complete the two-way tables and use them to write the sample space for all the possible outcomes.

a Flicking two light switches.

		Switch 2	
		On	Off
Switch 1	On		
	Off		

$S =$

b Tossing a coin and rolling a four-sided die.

		4 sided die			
		1	2	3	4
Coin	Head (H)				
	Tail (T)				

$S =$

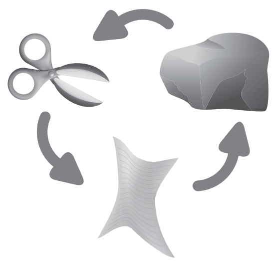
c Six friends logged onto a social website to chat.

		Friends					
		Ari (A)	Coco (C)	Qian (Q)	Jee Un (J)	Steve (S)	Fahim (F)
Logged on	Yes (Y)						
	No (N)						

$S =$

d Two people playing Scissors, Paper, Rock.

		Player 2		
		Scissors (S)	Paper (P)	Rock (R)
Player 1	Scissors (S)			
	Paper (P)			
	Rock (R)			

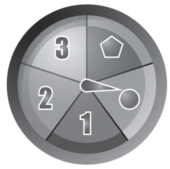


$S =$

Sample space

e The spinner and six-sided die pictured used together:

		Spinner				
		1	2	3	●	⬟
Die	1					
	2					
	3					
	4					
	5					
	6					



$S =$

3 Here is your first chance to earn an awesome passport stamp.

You hear someone say:

> "I do not have a twelve-sided die, but I do have two six-sided dice which we could just add together, which is the same thing."

=

?

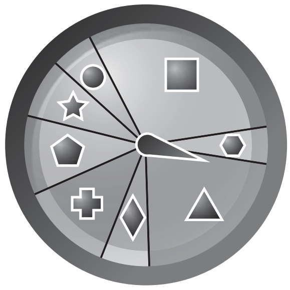
Use your understanding of sample space for both options to explain why you agree or disagree with this statement. Show all your working to back up your explanation.

Equally likely outcomes

Outcomes with the same chance of occuring as each other are called equally likely outcomes.

For these two spinners, the sample space of possible outcomes is exactly the same.

S = {★, ●, ■, ⬣, ▲, ♦, ✚, ⬟}



All segments are the same size.

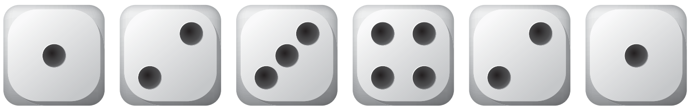
∴ The outcomes here are equally likely.

All segments have different sizes

∴ The outcomes here are not equally likely.

Say whether or not these experiments have equally likely outcomes

(i) Rolling a six-sided die with the following values on each side:



$$S = \{1, 1, 2, 2, 3, 4\}$$

There are more chances for an outcome of 1 or 2, so **different** outcomes are not equally likely.

(ii) Opening a twenty page newspaper to an even numbered page.

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

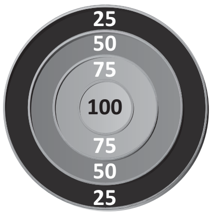
There is an equal number of odd and even page outcomes, so the outcomes are equally likely.

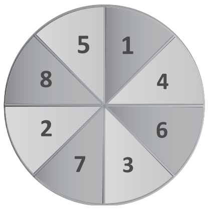
Some experiments can be ‘rigged’ to ensure certain outcomes. This is called bias.
Biased experiments do not have equally likely outcomes.
For example:

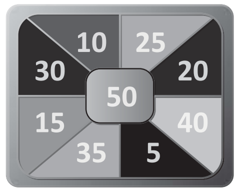
- The spinner above on the right is biased to give a result of ■ or ▲ each time.
- A normal six-sided die is weighted on one side to favour the number 6 getting rolled each time.

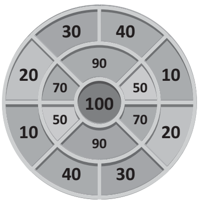
Equally likely outcomes

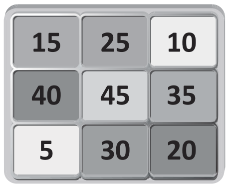
1. Points in dart games depend on what area of the board each dart lands. Label each board as having scores that are 'equally likely' or 'not Equally likely' with each dart thrown.

a 

b 

c 

d 

e 

f 

2. Write down the sample space for each of these events and state whether each different outcome is equally likely or not equally likely.

a Tossing a head or a tail on a normal coin.

$S =$ Each different outcome is:

b Picking a coloured marble from a bag containing three black and two green marbles.

$S =$ Each different outcome is:

c A raffle in which each of the ten participants have one ticket only.

$S =$ Each different outcome is:

d Picking a vowel or consonant from a bag containing all the letters from A to J.

$S =$ Each different outcome is:

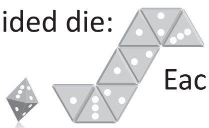
e Hitting an odd or even number when throwing a dart at the board in Q1 e).

$S =$ Each different outcome is:

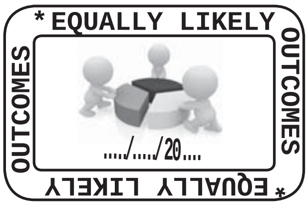
f Rolling an odd or even number on a four-sided die containing the first 4 positive perfect cubes.

$S =$ Each different outcome is:

g Rolling the following eight-sided die:



$S =$ Each different outcome is:

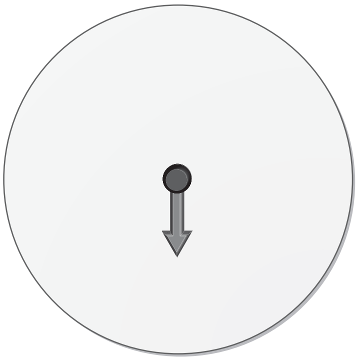


Equally likely outcomes

3 Use the given sample space and rule to fill in each of these blank spinners:

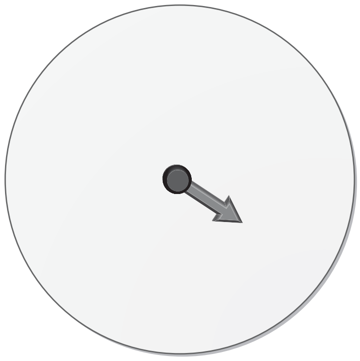
a $S = \{1, 2, 3, 4\}$

All outcomes equally likely



b $S = \{2, 4, 6, 8, 10\}$

All outcomes are not equally likely

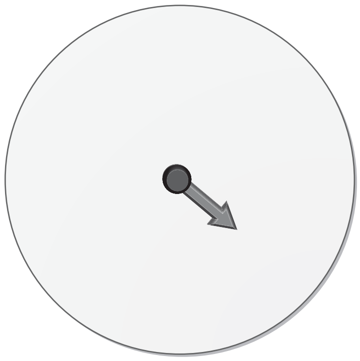


c $S = \{$●, ■, ▲, ♦, ⬟$\}$

Four sided shapes are equally likely. All the other shapes are not equally likely.

d $S = \{$Green, Green, Blue, Blue, Black, Black, Pink, Pink$\}$

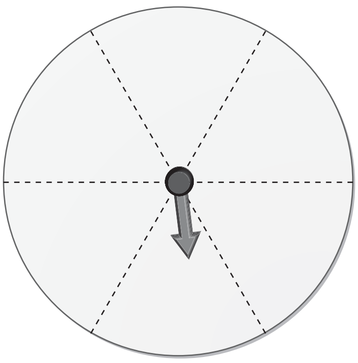
All outcomes are equally likely to occur.



e $S = \{$●, ■, ▲, $1, 2\}$

The spinner is biased so the circle outcome is twice as likely as the other four, equally likely outcomes.

Psst!: the spinner has already been divided up in a way to help you with this one.



f $S = \{1, 2, 3\}$

The spinner is biased so that each outcome has double the chance of being spun than the previous one.

Psst!: If the outcome 1 has one chance, then the outcome 2 has two chances etc!

Chance experiments

Chance experiments (called trials) investigate how often events occur through testing and observation.

The experimental probability is the value we get when the event we are looking for (favourable outcome) is compared to the total number of trials.

Number of times it happens

$$\text{Experimental probability of an event } (E) = \frac{\text{The frequency of the event } (E)}{\text{The number of trials completed}}$$

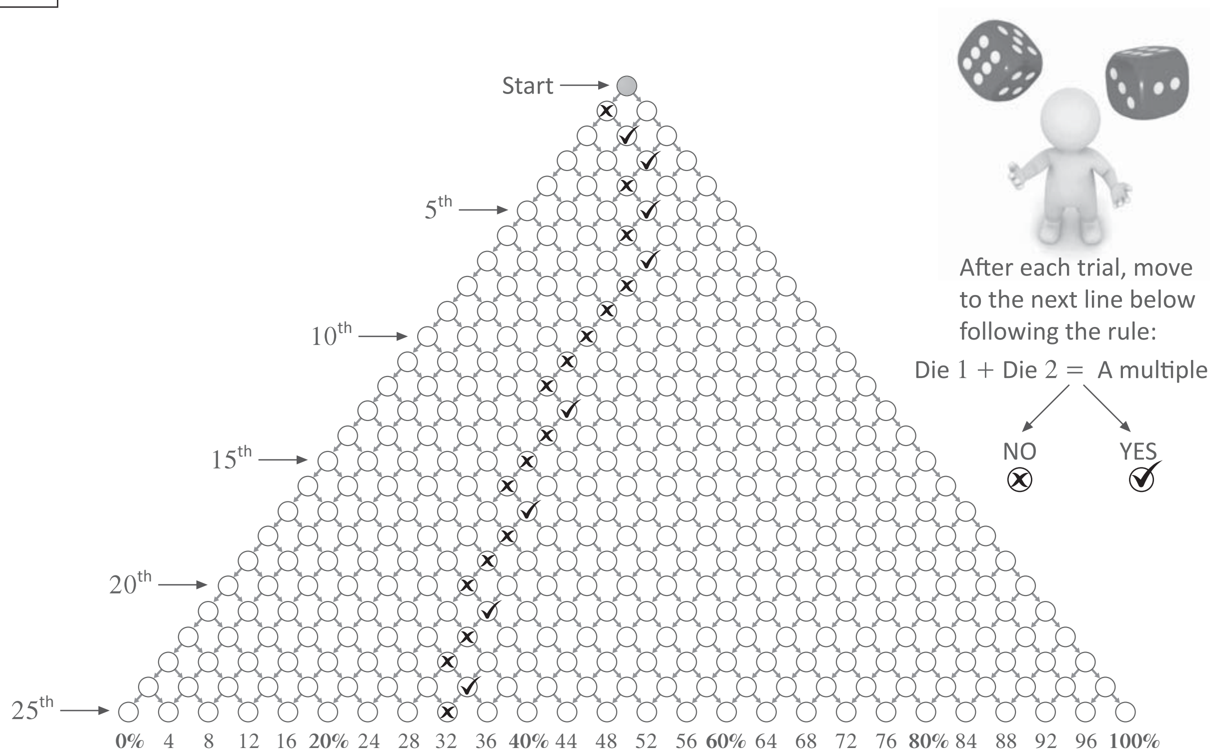
The more times each trial is repeated, the more accurate the experimental probability value will be.

Here is one way repeated trials can be recorded to approximate the experimental probability.

Record each result using the triangle to see the experimental chance of getting the favourable outcome

Two dice were rolled 25 times to see how often their sum was a multiple of three. The favourable outcomes in the results are circled.

Die 1:	6,	5,	3,	2,	3,	2,	2,	6,	3,	2,	4,	4,	3,	4,	1,	6,	2,	4,	5,	2,	1,	4,	1,	1,	6,
Die 2:	2,	1,	3,	5,	3,	5,	1,	1,	1,	6,	1,	4,	6,	3,	4,	5,	4,	6,	3,	2,	2,	6,	1,	2,	5,
$1 + 2$	8,	6,	6,	7,	6,	7,	3,	7,	4,	8,	5,	8,	9,	7,	5,	11,	6,	10,	8,	4,	3,	10,	2,	3,	11,



After each trial, move to the next line below following the rule:

Die 1 + Die 2 = A multiple of 3?

NO

YES

Experimental percentage probability

Using the formula,

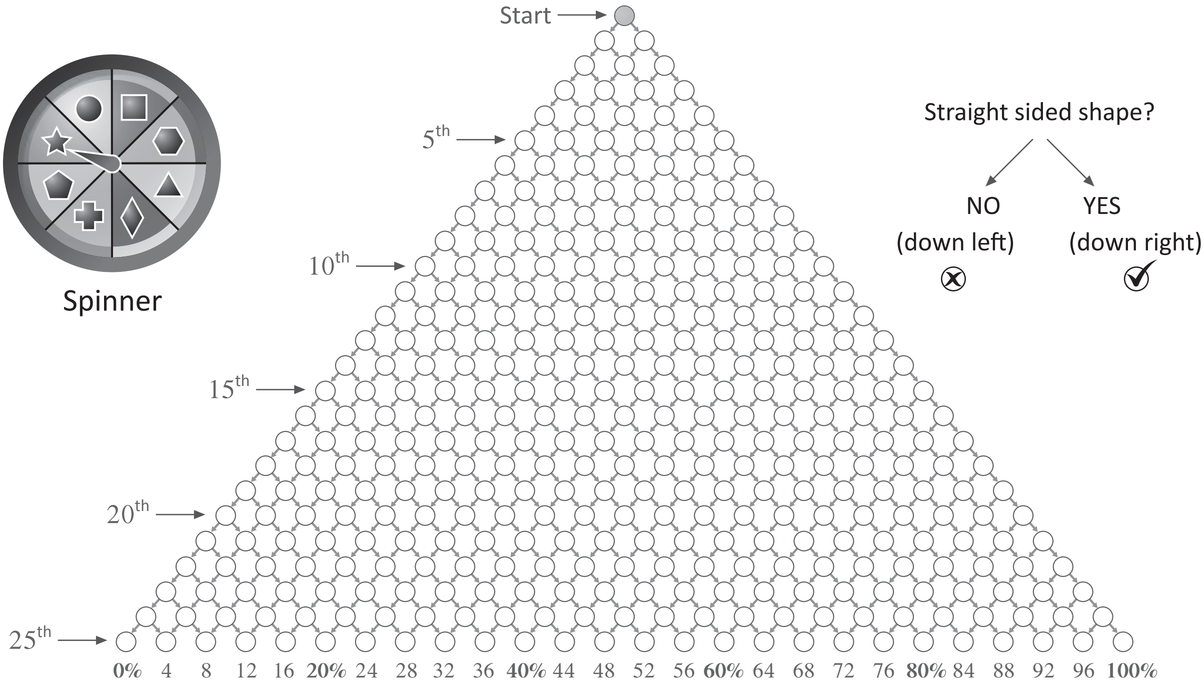
$$\text{Experimental probability} = \frac{\text{Frequency for the multiples of 3 rolled}}{\text{The total number of rolls made}} = \frac{8}{25} = \frac{32}{100} = 32\%$$

Chance experiments

1 The spinner shown below was used 25 times to see how often a shape with straight sides was spun.

Results: ✚, ⬟, ●, ■, ▲, ⬟, ⬢, ■, ★, ⬟, ●, ◆, ●, ▲, ✚, ★, ⬢, ●, ▲, ■, ⬟, ✚, ◆, ◆, ★.

a Fill in the triangle to find the experimental probability of spinning a favourable outcome.



Experimental probability (%)

Experimental probability = ______ %

b Use the formula to calculate the experimental probability from the results spun.

c Do you think this is accurate? Explain your answer, mentioning the possible outcomes on the spinner.

d What could you do to increase the accuracy of the experimental probability?

Theoretical probability

It compares how many times a particular event can happen with all the possible outcomes.

- Total favourable outcomes = the number of outcomes matching the result we are looking for.
- Total possible outcomes = the total number of outcomes in the sample space.

These terms are used a lot in probability

$$\text{Probability of an event} = \frac{\text{Total number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

The total number of favourable outcomes can never be more than the total number of possible outcomes

$\therefore$ The probability of an event can only be any value from 0 to 1.

$\therefore$ Probabilities in percentage form can only be any value from 0% to 100%.

Find the probability of these events

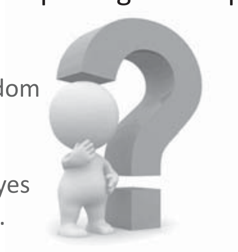
(i) Picking a red apple from a bowl containing 20 apples (12 red and 6 green).

$$\text{Probability of picking a red apple} = \frac{\text{The number of red apples (favourable outcomes)}}{\text{The total number of apples (possible outcomes)}}$$

$$= \frac{12}{20}$$

$$= \frac{3}{5} \quad \text{Simplify fraction}$$

Picking at random is like drawing from a hat or closing your eyes before picking.



So this means, for every five apples picked randomly from the bowl, three will probably be red apples!

(ii) The probability of rolling a 4 on a twenty-sided die, as a fraction, decimal and percentage

$$\text{Probability of rolling a } 4 = \frac{\text{The number of 4s on the die (favourable outcome)}}{\text{The total number of sides on the die (possible outcomes)}}$$

$$= \frac{1}{20} \quad \text{Fractional probability}$$

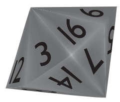
$$\therefore \frac{1}{20} = \frac{5}{100}$$

$$= 0.05 \quad \text{Decimal probability}$$

$$= 5\% \quad \text{Percentage probability}$$

Theoretical probability

4 If the sixteen-sided die pictured here is rolled, write the percentage probability of getting these outcomes:



a An even number.

b A multiple of four.

c A number between two and fifteen.

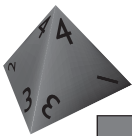
d A perfect square.

5 Fergus has twenty gaming application files on his phone. What is the decimal probability that when using these applications he is playing:

a One of the four strategy games?

b One of the twelve racing games?

6 The sample space table here shows the sum of two 4-sided dice when rolled together.



		Die 2			
	+	1	2	3	4
Die 1	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

a How many possible outcomes are there?

b How many different sums outcomes are there?

c Calculate the probability of rolling a total less than 5 as a simplified fraction.

d Calculate the decimal and percentage probability of rolling a prime number.

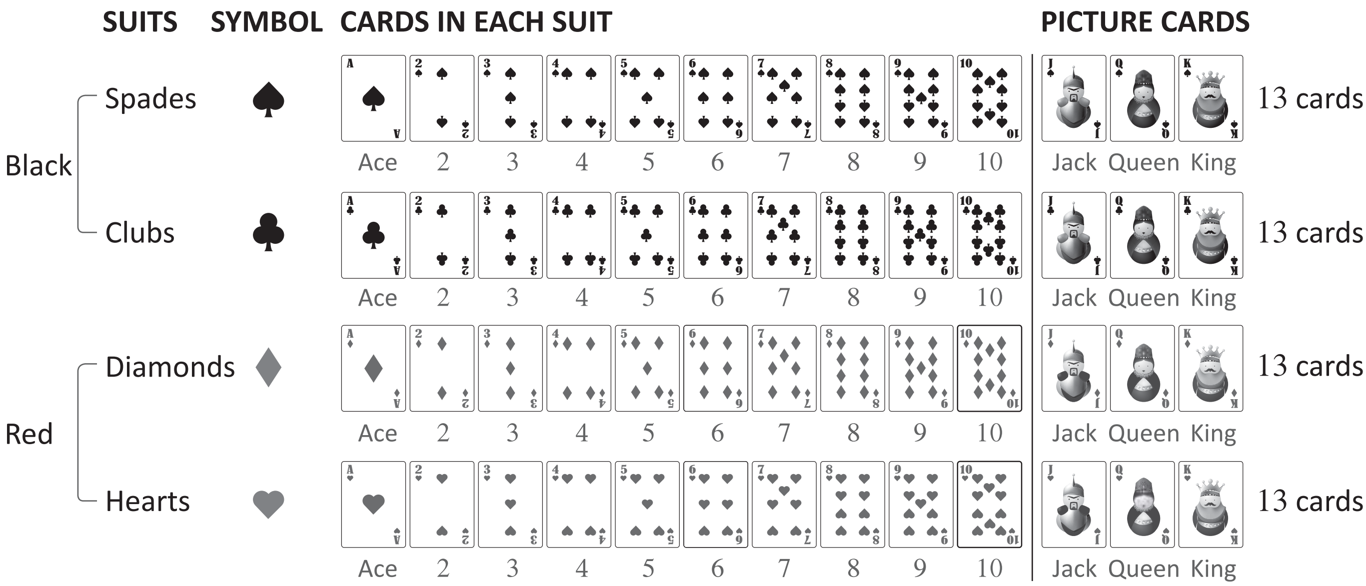
7 Explain in a sentence or two why it is not possible to have a probability greater than 100%.

Playing cards

While skill is required for many card games, they all require 'luck' to be on your side.

Luck is just chance working in your favour!

Here is what a normal pack of playing cards looks like all laid out (without the Joker cards).



Total = 52 cards

Playing cards are excellent for probability questions involving equally likely outcomes.

Calculate the probabilities of each of these

(i) Picking a heart card from a normal pack of playing cards.

$$P(\text{heart card}) = \frac{n(\text{heart cards})}{n(\text{cards in the pack})}$$

$$= \frac{13}{52}$$

$$= \frac{1}{4} \quad \text{Simplified fraction}$$

(ii) Picking a picture card from a normal pack of playing cards

$$P(\text{picture card}) = \frac{n(\text{picture cards})}{n(\text{cards in the pack})}$$

$$= \frac{12}{52}$$

$$= \frac{3}{13} \quad \text{Simplified fraction}$$

Playing cards

Remember:
numerator ÷ denominator
for decimal value

1 For each of these:

(i) Write the probability as a decimal if one card was selected at random from a pack of normal playing cards.
(ii) Describe the chance of each event.

a $P(\text{red card})$

(i) (ii)

b $P(\text{club})$

(i) (ii)

c $P(\text{black diamond})$

(i) (ii)

d $P(\text{Ace})$

(i) (ii)

e $P(\text{King of spades})$

(i) (ii)

2 The same words can be used to describe the chance of picking an Ace or picking a King of Spades.
However the calculated probabilities of each event are different.
How would you describe the chance of both events if comparing the two?