1. In these linear equations the variable appears on boths sides. Solve for the missing value:

a $2u - 10 = 3u$

b $7x - 18 = 3x + 10$

c $3(x + 2) = -3$

d $4y + 18 = 12 - 2y$

e $10(n - 6) = 2(10 + n)$

f $6m - 4 = -5(m + 3)$

g $8(k - 4) - 5k + 3 = 4$

h $5(2y - 1) - 6(y - 2) + 3 = 6$

i $8t - (2t - 18) = -12$

j $2(a + 3) - 3(a + 4) = -10$

2. Find Ivan's mistake when he tried to solve this equation?

$$3(h + 2) = 2(h + 1) + 5$$
$$3h + 2 = 2h + 2 + 5$$
$$3h + 2 = 2h + 7$$
$$3h + 2 - 2h - 2 = 2h + 7 - 2h - 2$$
$$h = 5$$

3. Solve these linear equations which contain fractions:

a $\frac{x}{8} - 1 = 4$

b $\frac{n}{2} + 4 = \frac{5n}{12} + 5$

c $\frac{3b + 4}{5} = -4$

d $c + \frac{c}{2} = 12$

e $\frac{16r + 2}{5} = 10$

f $\frac{m}{2} + \frac{m}{3} = 5$

4. Solve these linear equations which contain fractions:

a $\frac{7q+5}{3} = \frac{4q-30}{6}$

b $\frac{5u+5}{6} + \frac{5u+10}{9} + 2 = 7$

c $\frac{2g}{3} + \frac{g}{10} = \frac{1}{2} + \frac{3g}{4}$

(Hint: Find LCD of ALL fractions)

d $2 = \frac{6}{x}$

(Hint: Multiply both sides by the denominator)

e $\frac{6}{8d-2} = 1$

f $\frac{3k-45}{k} = 2$

5. Three times a number is 45. What is the number?

6. Claire, Leanne and Lindsay are sisters. Claire is two years older than Leanne and Leanne is 4 years older than Lindsay. The sum of all their ages is 54. How old is each sister?

7. Charlie has been collecting stamps which he keeps in two separate books. The second book has 7 more than triple the stamps of the first book. If he has 35 stamps in total (from both books) then:

a How many stamps are in the first book?

b How many stamps are in the second book?

8. Victor has a bag filled with 2c and 5c coins. He has 2 more coins worth 2c than the coins worth 5c. How many 2c coins does Victor have if all his coins sum to 88c?

What are Linear Inequalities?

An inequality is a mathematical expression with two sides separated by one of these inequality signs:

- $>$ greater than
- $\geq$ greater than or equal to
- $<$ less than
- $\leq$ less than or equal to

For example $3 < 4$ and $8 > 2$.

If there is a variable in the inequality and its highest power is 1, then the inequality is a linear inequality (unless the variable is in a denominator). These expressions are all linear inequalities because an inequality appears in each of them and the highest power of the variable is 1.

a $x + 2 < 5$ b $3x \geq 18$ c $-3x \geq 18$ d $\frac{x}{4} < 7$ e $2(x - 4) \leq 16$

How are Inequalities Solved?

Just like equations, the aim is to simplify the inequality to get the variable by itself on one side. Whatever is done to one side must be done to both sides.

Solve these inequalities

a
$$y + 3 \geq 10$$
$$y + 3 - 3 \geq 10 - 3$$
$$y \geq 7$$

b
$$3m - 4 < 14$$
$$3m - 4 + 4 < 14 + 4$$
$$3m < 18$$
$$\frac{3m}{3} < \frac{18}{3}$$
$$m < 6$$

Multiplying or Dividing by a Negative Number

Everyone knows that $2 < 5$ is true. If both sides are multiplied by -1 then $-2 < -5$. This is NOT true. If both sides of an inequality are multiplied by a negative number then the inequality sign must be reversed. So $-2 > -5$.

Solve these inequalities

a
$$-\frac{x}{2} > 5$$
$$2 \times -\frac{x}{2} > 2 \times 5$$
$$-x > 10$$
$$x < -10$$

Inequality sign is reversed after multiplying both sides by -1

b
$$5 - 3x \leq 17 - 5$$
$$5 - 3x - 5 \leq 17 - 5$$
$$-3x \leq 12$$
$$x \geq -4$$

Inequality sign is reversed after multiplying both sides by -3

Other than reversing the inequality sign when multiplying or dividing by a negative number, inequalities are solved in the same way as equations.

Variable is on both sides

a

$$5x + 25 \geq 2x + 4$$
$$5x + 25 - 25 \geq 2x + 4 - 25$$
$$5x - 2x \geq 2x - 21 - 2x$$
$$\frac{3x}{3} \geq \frac{-21}{3}$$
$$x \geq -7$$

b

$$3m + 3 < 5m - 5$$
$$3m + 3 - 3 < 5m - 5 - 3$$
$$3m - 5m < 5m - 8 - 5m$$
$$-2m < -8$$
$$m > 4$$

Inequality sign is reversed after dividing both sides by -2

Inequalities with brackets

a

$$3(q + 3) \leq 27$$
$$3q + 9 \leq 27$$
$$3q + 9 - 9 \leq 27 - 9$$
$$\frac{3q}{3} \leq \frac{18}{3}$$
$$x \leq 6$$

b

$$2(y + 5) > 3(y + 5)$$
$$2y + 10 > 3y + 15$$
$$2y + 10 - 10 > 3y + 15 - 10$$
$$2y - 3y > 3y + 5 - 3y$$
$$-y > 5$$
$$y < -5$$

Inequality sign is reversed after dividing both sides by -1

Inequalities with fractions (multiply by the LCD of ALL the fractions in the inequality)

a

$$\frac{6k - 8}{5} \geq 8$$
$$5 \times \frac{6k - 8}{5} \geq 8 \times 5$$
$$6k - 8 + 8 \geq 40 + 8$$
$$\frac{6k}{6} \geq \frac{48}{6}$$
$$k \geq 8$$

b

$$\frac{t}{4} - \frac{2t}{3} \leq -10$$
$$12 \times \left(\frac{t}{4} - \frac{2t}{3}\right) \leq -10 \times 12$$

LCD of the fractions

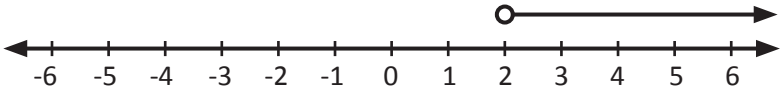
$$3t - 8t \leq -120$$
$$-5t \leq -120$$
$$t \geq 24$$

Inequality sign is reversed after dividing both sides by -4

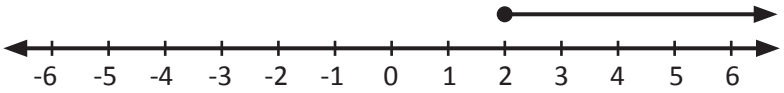
Graphing Inequalities

Solutions to inequalities can be represented on a number line.
For example, look at the inequality $x > 2$. This means x can be any number greater than, but not equal to 2.

On a number line $x > 2$ looks like this:



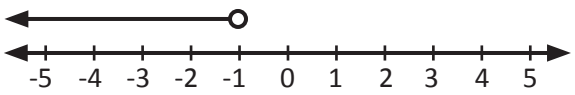
On a number line $x \geq 2$ looks like this:



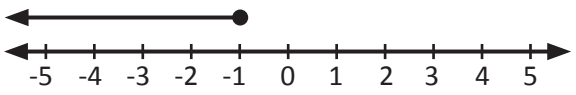
Can you spot the difference in the graphs above? In the first graph 2 is not included in the inequality (>), so the circle on the number line is hollow. In the second graph the inequality includes the number 2 (≥) so the circle is solid.

Here are some more examples:

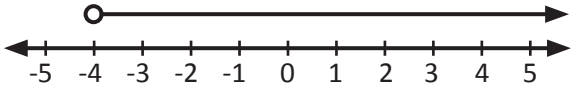
a $x < -1$



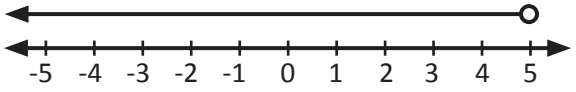
b $x \leq -1$



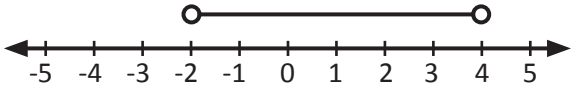
c $x > -4$



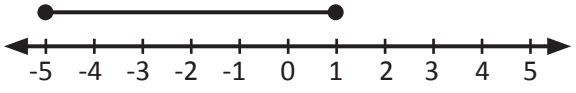
d $x < 5$



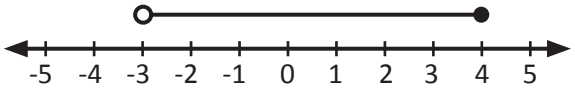
e $-2 < x < 4$



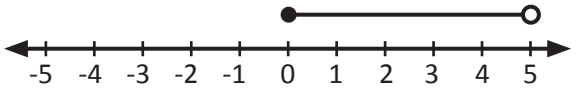
f $-5 \leq x \leq 1$



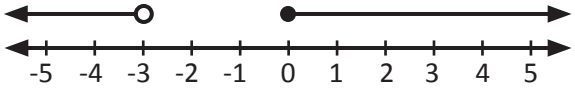
g $-3 < x \leq 4$



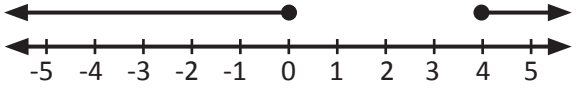
h $0 \leq x < 5$



i $x < -3$ or $x \geq 0$



j $x \leq 0$ or $x \geq 4$



1. Identify if the following are true or false:

a $6 > 3$

b $5 < 8$

c $3 > 8$

d $-2 < -5$

e $-4 > 4$

f $-8 < -4$

2. Solve these inequalities:

a $x + 3 < 4$

b $x - 4 \geq 5$

c $m + 7 \geq -4$

d $p - 10 \leq -8$

e $5q \leq 35$

f $4h + 3 > 51$

g $5x > 24 + x$

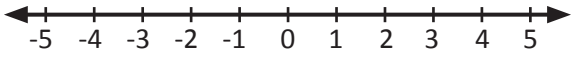
h $3(x - 1) \leq 3(1 - x)$

i $\frac{4h - 8}{2} < -2$

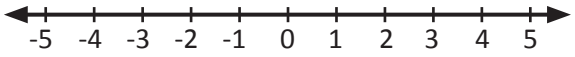
j $\frac{3x}{4} - \frac{2x}{5} \geq 14$

3. Graph these inequalities:

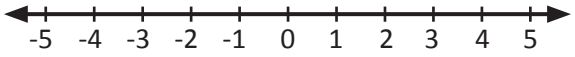
a $x > -3$



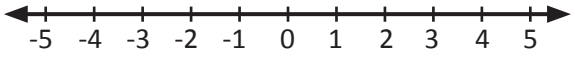
b $x \geq 2$



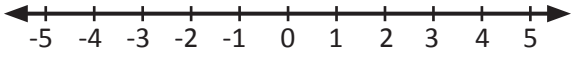
c $x \leq 0$



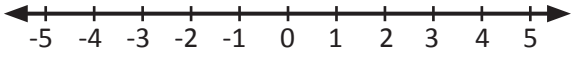
d $-1 < x < 3$



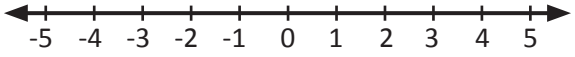
e $-5 \leq x < 5$



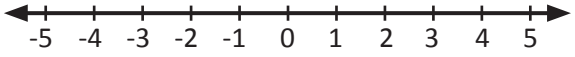
f $-2 < x \leq 4$



g $x < 0$ and $x \geq 3$

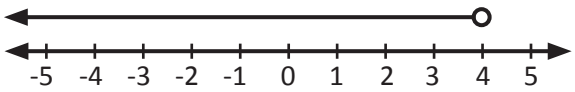


h $x > 4$ and $x \leq -1$

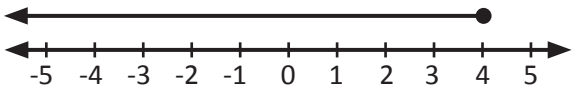


4. Write down the inequality represented by each of the following graphs:

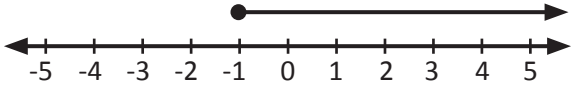
a



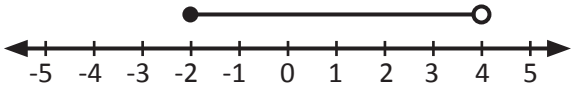
b



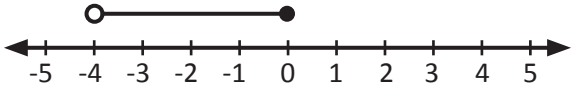
c



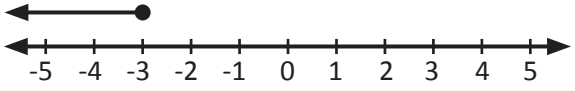
d



e



f



5. Solve these more complicated linear inequalities, then graph their solution:

a $5x + 6 > 9(x + 2)$

b $7y - 4(y + 4) > 5$

c $\frac{4 - 3d}{8} \leq 2$

d $\frac{b}{8} - \frac{b}{3} \leq -5$

e $\frac{a}{-2} \geq \frac{4a + 3}{10}$

f $\frac{6c - 2}{48} < \frac{1}{2} - \frac{c}{12}$

Find the solution to these simultaneous equations:

$2y + x = 3$ ① and $3y + 4x = 2$ ②

Using a substitution, b elimination and c graphical method.

a Substitution

Step 1: Make one of the variables the subject of ① $x = 3 - 2y$

Step 2: Substitute this expression into ② and solve:

$$\begin{aligned} 3y + 4(3 - 2y) &= 2 \\ 3y + 12 - 8y &= 2 \\ -5y &= -10 \\ y &= 2 \end{aligned}$$

Step 3: Substitute this value into ① or ② to solve for the remaining variable:

$$\begin{aligned} 2(2) + x &= 3 \\ x &= 3 - 4 = -1 \end{aligned}$$

So $x = -1$ and $y = 2$

b Elimination

Step 1: Make sure one of the variables has the same coefficient in both equations

$4 \times$ ① $= 8y + 4x = 12$ ③

③ has the same coefficient for x as ②

Step 2: Subtract equations with the same coefficients to eliminate a variable

$$\begin{array}{rl} 8y + 4x = 12 & \text{③} \\ -(3y + 4x = 2) & \text{②} \\ \hline 5y = 10 & \end{array}$$

So $5y = 10$

$\therefore y = 2$

Step 3: Substitute the value of the solved variable into any equation to find the value of the variable which is still unknown

Substitute $y = 2$ into ① to obtain:

$$\begin{aligned} 2(2) + x &= 3 \\ \therefore x &= 3 - 4 \\ x &= -1 \end{aligned}$$

So $x = -1$ and $y = 2$

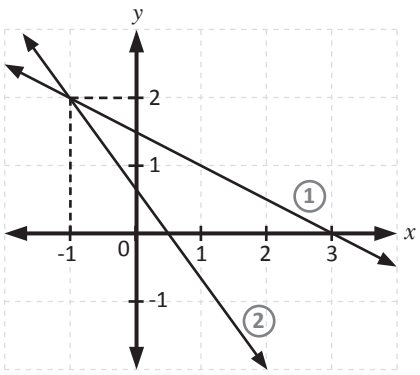
c Graphical

Step 1: Make y the subject of both equations

$$y = -\frac{1}{2}x + \frac{3}{2} \quad (1)$$

$$y = -\frac{4}{3}x + \frac{2}{3} \quad (2)$$

Step 2: Draw the graphs of these two equations on the same axes



Step 3: Read the point where the lines intersect

The lines intersect at $(-1, 2)$ so $x = -1$ and $y = 2$

As you can see, all three methods produce the same solution.

Simultaneous Equations Word Problems

As with single linear equations, word problems can be translated into simultaneous equations. Determine which TWO missing values are required and choose variables to represent these.

Write two equations using these variables and then use Substitution, Elimination or the Graphical Method to solve the equations.

The sum of two numbers is 12 and their difference is 6. Find the two numbers:

Let x and y represent the numbers.

So $x + y = 12$ (1)

and $x - y = 6$ (2)

These simultaneous equations can be solved using subsitution, elimination or the graphical method.

Juan is twelve years older than his sister Jamila. In two years Jamila will be half Juan's age. Find Juan and Jamila's age:

Let x = Juan's age

Let y = Jamila's age

$x - y = 12$ (1) $x + 2 = \frac{y}{2}$ (2)

These are simultaneous equations which can be solved using subsitution, elimination or the graphical method.

1. Write down 2 possible solutions for the variables in these equations:

a $x + y = 4$

b $2a + b = 6$

c $3x - 4y = 10$

2. Solve for the variables in these simultaneous equations using the substitution method:

a $2x + y = -1$

$x - 2y = -4$

b $2p + 3q = 10$

$2q - 4p = 44$

3. Use the graphical method to solve for these equations:

a $3x + 2y = 2$

$2x - y = 6$

b $3y - 4x = 24$

$2y + 2x = 2$

4. Solve for the variables in these simultaneous equations using the elimination method:

a $3x - y = -15$

$y + 2x = 0$

b $b - 4a = -12$

$3a - 2b = -1$

8. Find equations and solve them for these word problems (using any method):

a The sum of two numbers is 12. The sum of the first number, and double the second number is 16. What are the numbers?

b Ari is three years older than Eric. In three years from now, Ari will be twice as old as Eric will be. How old are they now?

c A resturaunt sells two kinds of meals: pizza and pasta. A pizza costs $\$14$ and a pasta costs $\$10$. In a single day the resturaunt sold 79 meals. If they earned $\$994$ on this day, how many of each meal was sold?